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# Simple Array Beam-Shaping Using Phase-Only Adjustments

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# Simple Array Beam-Shaping Using Phase-Only Adjustments

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#### **Abstract**

Conventional beam-shaping for array antennas is accomplished via an amplitude-taper on the elemental radiators. It is well known that proper manipulation of the elemental phases can also shape the antenna far-field pattern. A fairly simple transformation from a desired amplitude-taper to a phase-taper can yield nearly equivalent results.

## Acknowledgements

The preparation of this report is the result of an unfunded research and development activity.

### **Contents**

Forewo	ord	6		
Classification				
1 Introduction & Background				
2 Discussion				
2.1	Simple One-Dimensional Development	9		
2.2	One-Dimensional Array with Preexisting Amplitude Taper	17		
2.3	Two-Dimensional Array with Alternating Phase Deviations	18		
2.4	Two-Dimensional Array with Random Alternating Phase Deviations	23		
3 Conclusions				
References				
Distribution				

#### **Foreword**

This report details the results of an academic study. It does not presently exemplify any modes, methodologies, or techniques employed by any operational system known to the author.

#### Classification

The specific mathematics and algorithms presented herein do not bear any release restrictions or distribution limitations.

This distribution limitations of this report are in accordance with the classification guidance detailed in the memorandum "Classification Guidance Recommendations for Sandia Radar Testbed Research and Development", DRAFT memorandum from Brett Remund (Deputy Director, RF Remote Sensing Systems, Electronic Systems Center) to Randy Bell (US Department of Energy, NA-22), February 23, 2004. Sandia has adopted this guidance where otherwise none has been given.

This report formalizes preexisting informal notes and other documentation on the subject matter herein.

#### 1 Introduction & Background

It is well-known that the far-field antenna pattern is essentially the Fourier Transform of the antenna's aperture illumination function. Consequently, tapering the amplitude of the aperture illumination as a function of aperture position can be employed to alter the far-field antenna pattern, or shape it, in particular to mitigate problematic sidelobes. This is common for antenna designs of all types.

However, imparting an amplitude taper to an antenna aperture can be problematic. In particular, for many array-type antennas, especially those that employ distributed power amplifiers, it remains difficult to vary the signal amplitude significantly across the aperture. This is because a power amplifier's efficiency is maximized when the power amplifier devices are operated in compression. In this state, signal amplification is very nonlinear, making precise signal level attenuation or tapering difficult. Antennas of this type include Active Electronically Steered Array (AESA) antennas.

In such cases where amplitude tapers are difficult to implement, properly manipulating the phase of the various radiating elements may sometimes be employed to similarly engender a desired far-field antenna pattern.

Phase-only control of antenna patterns has been richly reported in the literature. Herein we elaborate on a technique discussed by DeFord and Gandhi<sup>1</sup> where the signs of the phase of neighboring radiating elements are alternated.

While we will discuss antenna patterns herein based on aperture illumination, we will take the liberty of doing so somewhat simplistically; ignoring some typically pesky details like mutual coupling between elements, elemental patterns, etc.

"Do what you can, with what you have, where you are."
-- Theodore Roosevelt

#### 2 Discussion

#### 2.1 Simple One-Dimensional Development

Consider a sampled amplitude taper which we define as

$$w(n)$$
 = aperture taper function, (1)

where

$$n =$$
aperture position index, with  $0 \le n \le N - 1$ . (2)

For convenience, we will assume *N* is even and elemental spacing is constant. A typical taper function might be a Hamming window, or perhaps a Taylor window. Window taper functions abound in the literature. An equally-spaced array with these amplitude weights as an aperture weighting will yield a far-field antenna pattern given by

$$G_{w}(\theta) = \sum_{n=0}^{N-1} w(n) e^{-j\frac{2\pi b}{\lambda} n \sin \theta} , \qquad (3)$$

where

 $\lambda$  = wavelength,

b = element spacing, and

$$\theta$$
 = Direction of Arrival (DOA) angle. (4)

The overall aperture length is calculated as

$$L_{ap} = b N. (5)$$

We now define a vector of unitary-amplitude values, but varying phase as

$$p(n) = e^{j\phi(n)} \text{ for } 0 \le n \le N - 1.$$
(6)

We relate the phase  $\phi(n)$  weighting to the amplitude taper with

$$\operatorname{Re}\left\{p(n)\right\} = \cos\left(\phi(n)\right) = w(n). \tag{7}$$

Of course, this formulation retains an ambiguity as to the sign of phase  $\phi(n)$ . That is, we must acknowledge that

$$\cos(\phi(n)) = \cos(-\phi(n)) = w(n). \tag{8}$$

We may embody this by creating a second vector composed of values  $\pm 1$  which we identify with

$$s(n) = \text{elements of a vector of } \pm 1 \text{ values.}$$
 (9)

This allows us to specify the sign of the phase as

$$\operatorname{sgn}(\phi(n)) = s(n). \tag{10}$$

We further identify that for most typical aperture weighting functions,  $|\phi(n)|$  will be fairly slowly varying, regardless of s(n). Now consider the far-field pattern of an aperture weighting given by p(n). This can be written as

$$G_p(\theta) = \sum_{n=0}^{N-1} p(n)e^{-j\frac{2\pi b}{\lambda}n\sin\theta} . \tag{11}$$

We may expand this to

$$G_{p}(\theta) = \sum_{n=0}^{N-1} \left[ \operatorname{Re} \left\{ p(n) \right\} + j \operatorname{Im} \left\{ p(n) \right\} \right] e^{-j\frac{2\pi b}{\lambda} n \sin \theta} , \qquad (12)$$

and recognizing Eqs. (7), (3), and (10), further expand this to

$$G_p(\theta) = G_w(\theta) + j \sum_{n=0}^{N-1} \left[ s(n) \sin(|\phi(n)|) \right] e^{-j\frac{2\pi b}{\lambda} n \sin \theta} . \tag{13}$$

For convenience, we identify the second 'imaginary' term as

$$G_m(\theta) = j \sum_{n=0}^{N-1} \left[ s(n) \sin(|\phi(n)|) \right] e^{-j\frac{2\pi b}{\lambda} n \sin \theta} . \tag{14}$$

Consequently, we may now write the far-field pattern of p(n) as

$$G_p(\theta) = G_w(\theta) + G_m(\theta) . \tag{15}$$

Since we expect  $\sin(|\phi(n)|)$  to be slowly-varying, we observe that s(n) may be chosen to modulate the pattern  $G_m(\theta)$  away from the desired response  $G_w(\theta)$ . A convenient

set of s(n) to do this is of the form of an alternating sequence of  $\pm 1$  values with an additional phase change at the center of the aperture. For example, for N=12 we would create the sequence where s(n) are elements of the vector

$$\mathbf{s} = \begin{bmatrix} -1, +1, -1, +1, -1, +1, -1, +1, -1, +1, -1 \end{bmatrix}. \tag{16}$$

Note the double +1 occurrence at the center of the array (designated with the color red). This maintains an even symmetry to the magnitude pattern of the phase-only pattern.

Element spacing requirements for such a phase-only pattern are somewhat more constraining than for an amplitude-weighted aperture. We illustrate with a sequence of examples. All examples will exhibit the following common parameters.

$$w(n) = -30$$
 dB Taylor weighting with  $\overline{n} = 5$ ,  
 $L_{ap} = 1$  m,  
 $\lambda = 0.02$  m, and  
 $s(n) = \text{as described by Eq. (16)}.$  (17)

#### **Example 1:** $b = \lambda/2$

Figure 1 illustrates the phase-only pattern  $G_p(\theta)$  and compares it to the amplitude-tapered reference pattern. Figure 2 illustrates the constituent patterns  $G_w(\theta)$  and  $G_m(\theta)$ . We observe that the undesired grating lobes in  $G_p(\theta)$  are visible, and are due to the contribution of the imaginary term  $G_m(\theta)$ . These can be pushed out of the visible region of the pattern by reducing the element spacing b to something finer.

#### **Example 2:** $b = \lambda/4$

Figure 3 illustrates the phase-only pattern  $G_p(\theta)$  and compares it to the amplitude-tapered reference pattern. Figure 4 illustrates the constituent patterns  $G_w(\theta)$  and  $G_m(\theta)$ . The undesired grating lobes in  $G_p(\theta)$  have been pushed out of the visible region.

#### Example 3: $b = \lambda/8$

Figure 5 illustrates the phase-only pattern  $G_p(\theta)$  and compares it to the amplitude-tapered reference pattern. Figure 6 illustrates the constituent patterns  $G_w(\theta)$  and  $G_m(\theta)$ . The undesired grating lobes in  $G_p(\theta)$  have been pushed even farther out of the visible region.

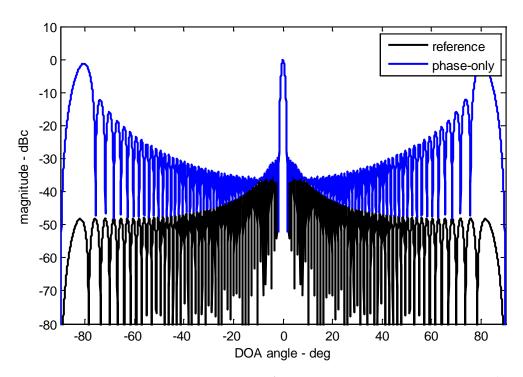


Figure 1. Beam patterns for element spacing of  $\lambda/2$  and -30 dB Taylor weighting ( $\overline{n}=5$ ). The reference pattern is the result of conventional amplitude weighting. The 'phase-only' pattern uses the phase sign sequence exemplified in Eq. (16).

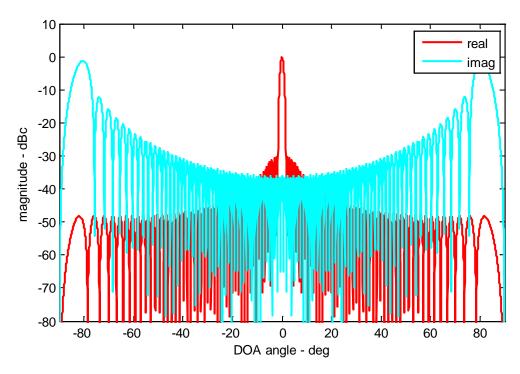


Figure 2. Real and imaginary components  $G_{\scriptscriptstyle W} \big( \theta \big)$  and  $G_{m} \big( \theta \big)$  respectively.

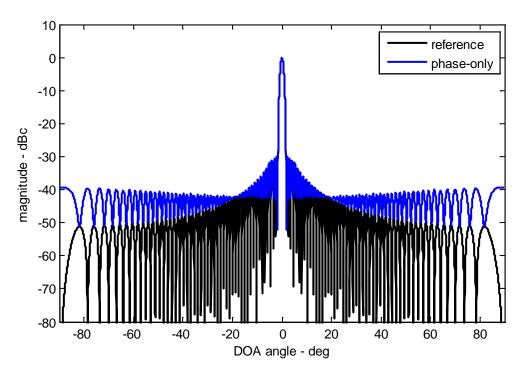


Figure 3. Beam patterns for element spacing of  $\lambda/4$  and -30 dB Taylor weighting ( $\overline{n}=5$ ). The reference pattern is the result of conventional amplitude weighting. The 'phase-only' pattern uses the phase sign sequence exemplified in Eq. (16).

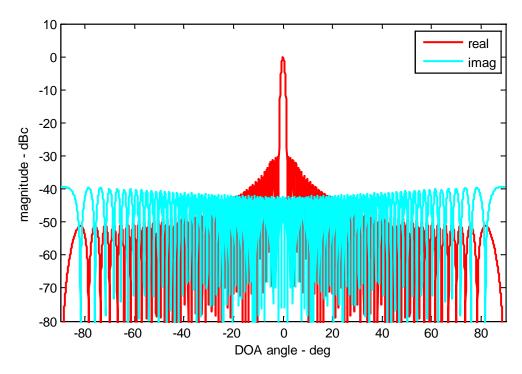


Figure 4. Real and imaginary components  $G_{\scriptscriptstyle W} ig( heta ig)$  and  $G_{m} ig( heta ig)$  respectively.

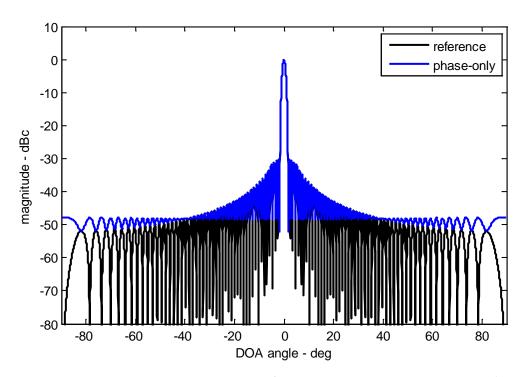


Figure 5. Beam patterns for element spacing of  $\lambda/8$  and -30 dB Taylor weighting ( $\overline{n}=5$ ). The reference pattern is the result of conventional amplitude weighting. The 'phase-only' pattern uses the phase sign sequence exemplified in Eq. (16).

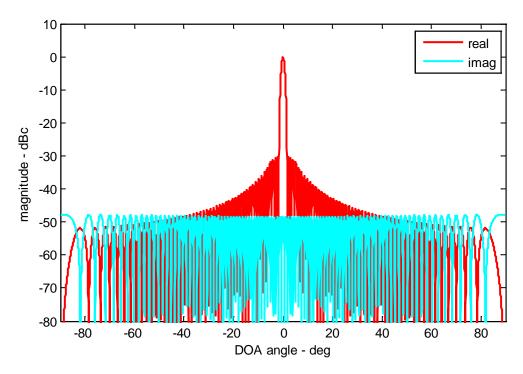


Figure 6. Real and imaginary components  $G_{\scriptscriptstyle W} ig( heta ig)$  and  $G_{m} ig( heta ig)$  respectively.

#### **Comments**

We offer the following comments.

- The mainlobe of the phase-only beam patterns are virtually identical to the amplitude-only beam patterns. The differences are in the sidelobes.
- Element spacing for phase-only weights will generally require a finer element spacing than for amplitude-only weights.
- DeFord and Gandhi suggest that for the alternating sequence of phase signs, "For small element spacing, of less than about  $3\lambda/8$ , the contribution to the radiation pattern from the imaginary part of the excitations may be almost completely removed from the visible region of the spectrum."
- The element spacing issue is exacerbated by steering the beam away from boresight (normal) to the array. This makes visible portions of the beam pattern not otherwise visible for broadside patterns. Figure 7 illustrates the case of a beam steered 40 degrees away from boresight.
- Although we mention it here now, we have not addressed the effects of phase quantization, nor will we in this report.

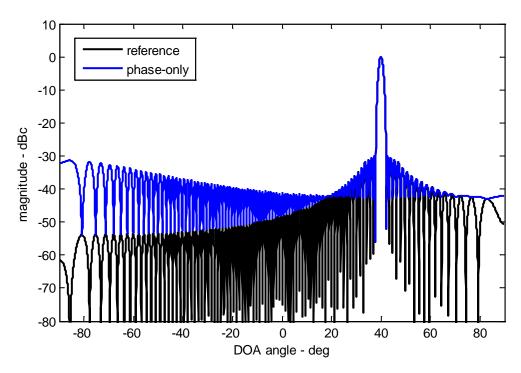


Figure 7. Beam patterns for element spacing of  $\lambda/4$  and -30 dB Taylor weighting ( $\overline{n}=5$ ). The beam is steered to 40 degrees. The reference pattern is the result of conventional amplitude weighting. The 'phase-only' pattern uses the phase sign sequence exemplified in Eq. (16).

#### **A Note About Element Spacing**

A question arises as to whether array element spacing can be practically achieved to take advantage of the phase-only weighting discussed in the previous sections.

The short answer is "Yes."

We note that metamaterials are a relatively recent development that allows substantial shrinking of the antenna element dimensions, including their spacing in an array, from the common metric of half the free-space wavelength. Much literature on this topic has been published, where shrinking factors of up to high-single-digits have been frequently reported, and sometimes more. Representative papers include those by Buell, et al., Mosallaei and Sarabandi, and Bilotti, et al.

#### 2.2 One-Dimensional Array with Preexisting Amplitude Taper

The preceding development tried to match a desired precise amplitude-taper with a calculated phase-taper applied to an otherwise actual uniform amplitude-taper. That is, the actual amplitude-taper of the array was uniform, and beam-shaping was done with purely a phase function. We now relax the stipulation of presuming an actual uniform amplitude-taper. Accordingly, we define and distinguish

$$w(n)$$
 = aperture amplitude-taper function that we desire to match, and  $w_{pre}(n)$  = preexisting amplitude-taper that we actually have to start with. (18)

Our preceding development must therefore be modified to account for the fact that we have a preexisting amplitude-taper  $w_{pre}(n)$  that is not uniform. We do so by adjusting Eq. (6) to define the vector of element weights as

$$p(n) = w_{pre}(n)e^{j\phi(n)} \text{ for } 0 \le n \le N-1.$$

$$\tag{19}$$

Consequently, we now relate the phase to both of the amplitude-tapers with

$$\operatorname{Re}\left\{p(n)\right\} = w_{pre}(n)\cos(\phi(n)) = w(n),\tag{20}$$

from which we calculate the specific phase we now require with

$$\cos(\phi(n)) = \frac{w(n)}{w_{pre}(n)}.$$
(21)

Of course, this means that we must observe the constraint on amplitude-tapers

$$w(n) \le w_{pre}(n). \tag{22}$$

The sign of the phase may then be calculated as before.

This affects the undesired spectrum component, which is now calculated as

$$G_m(\theta) = j \sum_{n=0}^{N-1} \left[ s(n) w_{pre}(n) \sin(|\phi(n)|) \right] e^{-j\frac{2\pi b}{\lambda} n \sin \theta} . \tag{23}$$

If all  $w_{pre}(n) \le 1$ , with some actually less than unity, then we may expect a reduction in the energy in  $G_m(\theta)$ , which is desirable with respect to enhancing efficiency.

#### 2.3 Two-Dimensional Array with Alternating Phase Deviations

The analysis and techniques of the previous sections can be extended to two-dimensional apertures, i.e. two-dimensional arrays. Accordingly, we define a number of parameters for the two-dimensional aperture in terms of azimuth and elevation parameters.

Azimuth aperture parameters include

$$w_{az}(n)$$
 = aperture azimuth taper function,  
 $n$  = aperture azimuth position index, with  $0 \le n \le N-1$ , and  
 $b_{az}$  = element azimuth spacing. (24)

Elevation aperture parameters include

$$w_{el}(m)$$
 = aperture elevation taper function,  
 $m$  = aperture elevation position index, with  $0 \le m \le M - 1$ , and  
 $b_{el}$  = element elevation spacing. (25)

For convenience, we will assume M and N are even. Overall aperture lengths are calculated as

$$L_{ap,az} = b_{az} N$$
, and 
$$L_{ap,el} = b_{el} M$$
. (26)

We now define a matrix of unitary-amplitude values, but varying phase with the function

$$p(m,n) = e^{j\phi(m,n)}$$
 for  $0 \le m \le M - 1$  and  $0 \le n \le N - 1$ . (27)

We specify the sign of the phase as

$$\operatorname{sgn}(\phi(m,n)) = s(m,n). \tag{28}$$

We relate the phase to the amplitude taper with

$$\operatorname{Re}\left\{p(m,n)\right\} = \cos\left(\phi(m,n)\right) = w_{az}(n)w_{el}(m), \text{ and}$$

$$\operatorname{Im}\left\{p(m,n)\right\} = s(m,n)\left|\sin\left(\phi(m,n)\right)\right|. \tag{29}$$

We will assume a two-dimensional version of the alternating sequence previously illustrated in Eq. (16), which we exemplify with the following matrix for M = 8 and N = 12, for which s(m, n) are elements of **S**.

$$\mathbf{S} = \begin{bmatrix} +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 \\ -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 \\ -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 \\ -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 \end{bmatrix}.$$

$$(30)$$

We define a far-field antenna pattern geometry in Figure 8.

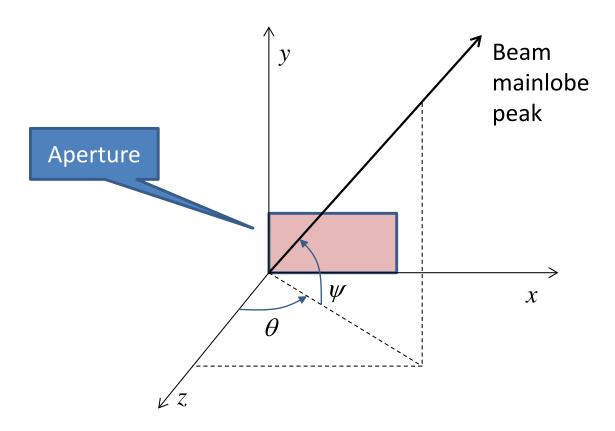


Figure 8. Geometry definitions for two-dimensional aperture.

The two-dimensional far-field antenna pattern of the phase-only taper function can be calculated as

$$G_p(\theta, \psi) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} p(m, n) e^{-j\frac{2\pi}{\lambda} \left(b_{az} n \cos \psi \sin \theta + b_{el} m \sin \psi\right)} . \tag{31}$$

The two-dimensional far-field antenna pattern of the reference amplitude taper function can be calculated as

$$G_{w}(\theta,\psi) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} w_{el}(m) w_{az}(n) e^{-j\frac{2\pi}{\lambda} \left(b_{az} n \cos \psi \sin \theta + b_{el} m \sin \psi\right)} . \tag{32}$$

We now illustrate with some examples. Subsequent examples, unless otherwise noted will use the following parameters.

$$w_{az}(n) = -30$$
 dB Taylor weighting with  $\overline{n} = 5$ ,  $w_{el}(m) = -30$  dB Taylor weighting with  $\overline{n} = 5$ ,  $L_{ap,az} = 0.50$  m,  $L_{ap,el} = 0.26$  m,  $\lambda = 0.02$  m, and  $s(m,n) = \text{as described by Eq. (30)}.$  (33)

#### **Example 4: Broadside DOA with** $b_{az} = \lambda/4$ , and $b_{el} = \lambda/4$

Figure 9 illustrates both the amplitude-tapered reference pattern  $G_w(\theta,\psi)$ , and the phase-only pattern  $G_p(\theta,\psi)$ . The beam is steered broadside to the aperture. Figure 10 illustrates projections of  $G_w(\theta,\psi)$  and  $G_p(\theta,\psi)$ , that is maximum values over azimuth and elevation angles. We observe that although small, undesired residual sidelobes are visible in the phase-only weighting pattern  $G_p(\theta,\psi)$ . These can be diminished even more by using finer element spacings  $b_{az}$  and  $b_{el}$ .

#### Example 5: Non-broadside DOA with $b_{az}=\lambda/4$ , and $b_{el}=\lambda/4$

Figure 11 and Figure 12 take the previous example and steer the beam to a -30 degree elevation offset and a -60 degree azimuth offset.

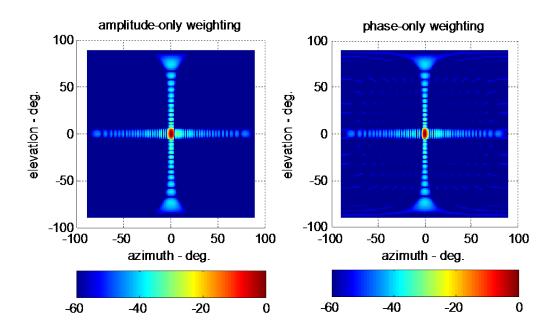


Figure 9. Two-dimensional far-field antenna patterns for (left) amplitude-only weighting, and (right) phase-only weighting, for broadside beam squint. Colors denote antenna gain in dBc.

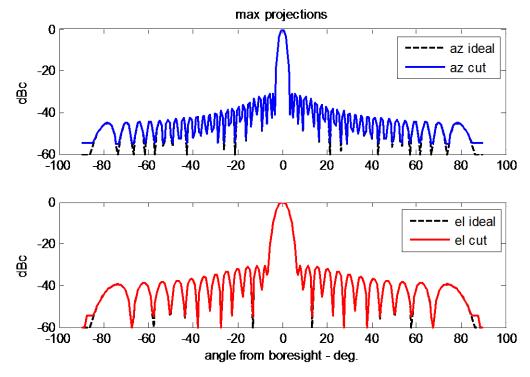


Figure 10. Projections of maximum antenna gain responses over (top) azimuth angle, and (right) elevation angle, for patterns in Figure 9.

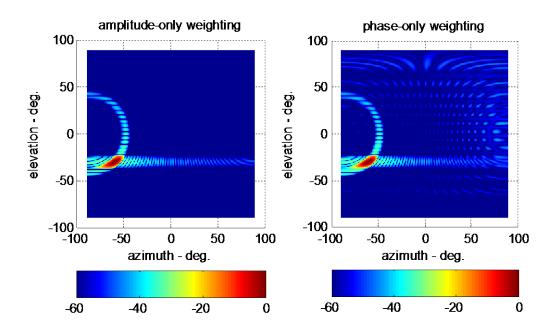


Figure 11. Two-dimensional far-field antenna patterns for (left) amplitude-only weighting, and (right) phase-only weighting, for non-broadside beam squint. Colors denote antenna gain in dBc.

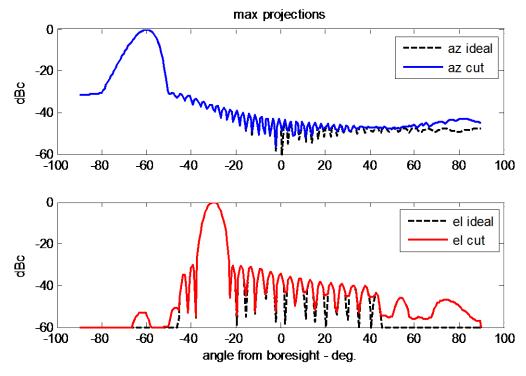


Figure 12. Projections of maximum antenna gain responses over (top) azimuth angle, and (right) elevation angle, for patterns in Figure 11.

# 2.4 Two-Dimensional Array with Random Alternating Phase Deviations

The previous development assumed that the sign of the phase, designated with s(m,n) was essentially an alternating sequence, albeit with even symmetry about the center of the array.

One might consider selecting the sign of the phase in some random fashion. We state without elaboration that simulations show that sidelobe levels are somewhat more problematic with such a selection algorithm for s(m,n).

"Between two evils, I always pick the one I never tried before."
-- Mae West

#### 3 Conclusions

We summarize herein the following key points.

- A phase-taper can yield a nearly equivalent far-field antenna pattern as an amplitude-taper.
- For such a phase-taper, with alternating signs on the elemental phases, grating lobes will manifest due to the alternating non-zero phases.
- Eliminating such grating lobes requires a somewhat finer spacing of the array elements than if amplitude-tapering is employed.
- Suitable finer element spacing is viable with metamaterial antennas.
- A preexisting amplitude-taper can be accommodated with a suitable phase-taper to yield a desired far-field pattern.
- Phase-tapers will work with both one-dimensional and two-dimensional arrays.
- Phase-tapers do not inhibit beam steering.

"The truth is rarely pure and never simple."
-- Oscar Wilde

#### References

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